Synergy Consulting

Rethinking Circularity in Project Finance Models

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Eliminating Circularity in Project Finance Models: The Coefficient Matrix Method

1 Introduction

Whether we are advising clients on raising debt or equity, supporting them through competitive bids and M&A of project assets, or assisting governments in tariff design, bid evaluation, and value-for-money assessment, every major decision in project finance ultimately traces back to the financial model.

One of the fundamental and persistent analytical challenges faced in project finance modelling is that of circularity. Circularity, very simply, occurs when a particular variable depends on another variable that, in turn, ultimately depends back on the first variable itself. This self-dependence creates a feedback structure within the model, where the output of a calculation is simultaneously one of its own inputs.

The often-quoted example is the determination of the Total Project Cost. The Total Project Cost includes Interest During Construction (IDC). IDC depends on the amount of Senior Debt, and the Senior Debt itself is calculated as a percentage of the Total Project Cost.

To handle this problem, there are two main approaches:

a) **Excel Iteration Method:**

Excel provides an iterative calculation option that can resolve circular references automatically. However, it is seldom used in practice because it can make the model unstable and difficult to control.

b) **Copy–Paste (Macro) Method:**

This is the more common method used in most project finance models. Here, the circular link is broken by copying a value and pasting it as a fixed number. A macro then keeps repeating this process until the two linked values converge. For example, the model may copy the latest calculated project cost, paste it as a value, and then re-run the calculations (based on the pasted value). The macro continues this copy–paste cycle until the copied (calculated) and pasted values of Project Cost differ by less than a pre-set tolerance. While this approach usually works, it has some drawbacks:

- For complex models with multiple scenarios (like P50/P90 in renewable projects) or refinancing options (Soft mini perms / balloon structures), several tariff optimizations etc. it can take long time to run. This can be especially challenging during the hours leading up to the bid submission, — a situation that served as the catalyst for developing the proposed new approach in this Article !
- It can be difficult for users (e.g, government and developer commercial teams) who are not conversant with macros / VBA to understand what the macro is doing in the background.

- It is often overused, with every (actual or perceived) circular link being pushed into the macro, which makes the model significantly slow.

Therefore, there is a need for a third approach that can overcome the limitations of the methods discussed above.

To that end, an examination of the circularity problem shows that once the circularity is solved, it leads to a unique outcome, implying the existence of a single, closed-form solution. To bring out this point, continuing with the same example:

Project Cost = EPC Cost + IDC

IDC = interest rate (i) × Debt Size, and Debt Size = Leverage (L) × Project Cost

Substituting these relationships:

Project Cost = EPC Cost + i × L × Project Cost

Rearranging terms:

Project Cost × (1 - iL) = EPC Cost

And solving for project cost gives:

Project Cost = EPC Cost / (1 - iL)

This shows that, for a simplistic case, instead of using iteration, we can arrive directly at a fixed mathematical solution.

Building on this observation, this article introduces a new approach to resolve circularities in project finance models. The relationships between variables are expressed in closed form equations, which can be represented and solved through a matrix of coefficients, hence the name Coefficient Matrix Method. This approach removes the need for macros or Excel iteration, thus providing a transparent and consistent (though mathematically more rigorous) way to handle circular links.

The following sections explain this Coefficient Matrix Method in detail.

2 Key Idea Behind the Coefficient Matrix Method

Circularity in project finance models most often appears during the construction phase. This is the period when the Project Cost is still building up and several components depend on each other. For instance, the Project Cost includes Interest During Construction (IDC), upfront fees, development fees, DSRA and commitment fees—all of which depend on the debt size (or directly on the Project Cost). At the same time, the debt size itself is calculated as a percentage of the total project cost. This circular relationship means that the Project Cost keeps shifting as these values are recalculated.

In contrast, the operations phase of a project usually has fewer and simpler circular relationships. Most circularities during this phase be handled with straightforward modeling techniques.

Therefore, the focus of this article is on the construction-phase circularities, which are both common and time-consuming to solve.

From a mathematical perspective, the Total Project Cost can be expressed simply as the sum of Senior Debt and Equity (in any form including EBLs), which funds it:

$$\text{Total Project Cost} = \text{Debt} + \text{Equity}.$$

So, the idea of Coefficient Matrix Method is to figure out a way to determine the Final Debt Size and Final Equity Size using closed form formulations. We will analyze these two components (Debt and Equity) separately. The next step is to look at how debt is sized. Once the method for determining debt is established, we can then identify the portion funded by equity, which is the more complex part. Together, these two components will define the overall Total Project Cost and allow us to express the entire system as a set of solvable relationships, with no circularity.

3 Calculating the Debt Component (Debt Sizing)

The first step is to determine how the debt amount is calculated within a project finance model. At any instant in the model (whether optimized or not) the tariff and cost assumptions are in place, i.e., the model will produce the Cash Flow Available for Debt Service (CFADS) for each period. This value represents the amount available to meet the scheduled loan payments.

Debt service for each period can be estimated by dividing CFADS by the Debt Service Coverage Ratio (DSCR) requirement. Hence, the debt service for any period t will be:

$$\text{Debt Service}_t = \text{CFADS}_t / \text{DSCR}$$

This gives the total payment (principal + interest) due to lenders in each period.

It is well established in fixed-income markets that if a series of future Debt Service payments is discounted at the cost of debt, the present value equals the initial debt amount. This is conceptually similar to valuing a bond: if you discount a bond’s coupon payments at the coupon rate, you obtain its face value. The same logic applies to project finance debt, regardless of whether the debt follows an amortizing structure or a bullet repayment. While this relationship is generally understood in practice, for the readers it is worth examining through a more rigorous mathematical derivation to appreciate the underlying intuition.

So, the initial debt size (D_0) can be written as:

$$D_0 = \Sigma\ [\text{Debt Service}_t / (1 + r_t)\text{\textasciicircum}t]$$

where r_t is the cost of debt (interest rate) applicable in period t, and n is the number of debt repayment periods.
This provides a direct way to calculate the Debt component that funds the Project Cost as mentioned in Section 2 without relying on macros, sculpting or iterative goal-seek methods.

This also resolves the issue of DSRA circularity as debt service for the period for which DSRA is to be sized at COD is known.

Few points to note, in most project finance models, debt sizing is typically carried out by specifying the target leverage as a percentage of total project cost and then assessing where the resulting DSCR settles. However, in most cases, the limitation on maximum debt capacity would be the DSCR requirement - i.e., the quantum of debt ultimately depends on the DSCR requirement rather than on the leverage input. The approach outlined here proposes to calculate the debt size directly from the DSCR requirements (which in almost all the cases would be the criteria determining the debt size). Furthermore, in some situations—particularly in post-bid or refinancing cases—the debt size is already fixed by lenders, in which case this debt-sizing step is not required.

Finance Income in Finance Lease and IFRIC 12 Cases

Taking a detour from Project Cost estimation, another area where modelling issues appear is in computing the interest rate (for finance income calculation)—particularly in Financial Lease arrangements. In such cases, practitioners typically attempt to "goal-seek" the rate that equates the present value of future lease or concession payments with the value of the financial asset at Commercial Operation Date (COD). This can be time consuming.

Instead of using macros for goal-seeking, this can be handled more efficiently using Excel's in-built XIRR function. The initial financial asset value at COD represents the investment, and the subsequent cash inflows (lease or service payments) represent the returns. The resulting IRR corresponds to the effective interest rate that satisfies the accounting requirement:

$$\text{Investment at COD} = \Sigma\ [\text{Lease Payment}_t / (1 + \text{r\_eff})^t]$$

It should be noted, however, that the IRR function assumes compounding of interest between periods. When applying this in financial models, care must be taken to ensure consistency between the compounding frequency used for the IRR and the periodicity of the cash flows.

By handling the Finance Income in this way, the model avoids goal seek macros. The same concept can be applied for projects where IFRIC 12 is applicable.

4 Calculating the Equity Component (Equity Sizing) – EBL Funding Only

To explain the principle of equity sizing, we start with a simple case where the Equity Bridge Loan (EBL) is the only source of project funding— that is, the project has no senior debt. Although this is not typical in practice, it serves as a useful starting point to understand the structure before adding further complexity.

Consider a project with a three-period construction phase: Period 1, Period 2, and Period 3 (the concept can be extended to any length of construction period). The following inputs are known:

Period	Hard Cost (H_n)	EBL Interest Rate (I_n)
1	H_1	I_1
2	H_2	I_2
3	H_3	I_3

Here, H_n represents the hard cost (EPC, development, or other known construction expenditure) in period n, and I_n is the applicable interest rate on the EBL for that period. Funding is assumed to be drawn at the end of each period, meaning that no interest accrues on the amount drawn within the same period.

Period 1

Project Cost for the Period = C_1; where

$C_1 = H_1$

$C_1 = 1 \cdot H_1 + 0 \cdot H_2 + 0 \cdot H_3$ (Equation X)

As funding is only through EBL,

$E_1 = C_1$

Period 2

Project Cost for the Period = C_2; where

$C_2 = H_2 + E_1 \cdot I_2$; as interest accrues on E_1

substituting and rearranging this gives

$C_2 = H_1 \cdot I_2 + H_2$

$C_2 = I_2 \cdot H_1 + 1 \cdot H_2 + 0 \cdot H_3$ (Equation Y)

Funding for the Period

$E_2 = C_2$

Period 3

Project Cost for the Period = C_3; where

$C_3 = H_3 + E_1 \cdot I_3 + E_2 \cdot I_3$; as interest accrues on E_1 and E_2

substituting and rearranging this gives

$C_3 = (I_3 + I_2 \cdot I_3) \cdot H_1 + I_3 \cdot H_2 + 1 \cdot H_3$ (Equation Z)

Funding for the Period

$E_3 = C_3$

These Periodic Project Costs are summarized in the table below

Period	Expression for Periodic Cost
1	$C_1 = 1 \cdot H_1 \quad + 0 \cdot H_2 \quad + 0 \cdot H_3$
2	$C_2 = I_2 \cdot H_1 \quad + 1 \cdot H_2 \quad + 0 \cdot H_3$
3	$C_3 = (1+ I_2)\, I_3 \cdot H_1 \quad + I_3 \cdot H_2 \quad + 1 \cdot H_3$

Further, they can be written in Matrix Form as follows: [C] = [I] x [H]; where

Periodic Costs [C]	Coefficient Matrix [CM]	Hard Costs matrix [H]

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ I_2 & 1 & 0 \\ I_3(1+I_2) & I_3 & 1 \end{bmatrix} \times \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix}$$

This can be generalized for ‘n’ periods, where each diagonal element of the Coefficient Matrix (**CM**) equals 1, representing the direct funding of that period’s own hard cost. Every cell below the diagonal corresponds to the interest for that period, multiplied by the cumulative sum of all coefficients above it. This can be summarized as below -

$$CM_{n,j} = \begin{cases} 1, & \text{if } n = j, \\ I_n \sum_{x=1}^{n-1} CM_{x,j}, & \text{if } n > j, \\ 0, & \text{if } n < j\ . \end{cases}$$

Where,

$CM_{n,j}$ = Any element of the Coefficient Matrix
n = row index
j = column index
x = summation index
I_n = EBL interest rate for period n

Because the Coefficient Matrix is structured so that each period only depends on the ones before it, the system can always be solved directly, producing one clear and consistent result for [C] based on the given hard costs [H]. Also, as EBL is the only source of funding

[E] = [C]; *where*

Periodic EBL Drawdown [E}		Periodic Costs [C]
$[E_1]$	=	$[C_1]$
$[E_2]$		$[C_2]$
$[E_3]$		$[C_3]$

Therefore, in effect, the EBL drawdown for each construction period is predetermined, which considers both i) funding of Hard costs and ii) interest during construction on the previously drawn EBL. As a result, when the schedule of sources and uses of funds is prepared, it will naturally align period by period, ensuring consistency between the EBL funds and the construction cost (including IDC on EBL). Thus, the issue of circularity does not arise — the model resolves itself mathematically rather than iteratively.

Extending the Logic to Other Cost Elements During Construction

This formulation can be extended to include other construction-phase financing costs associated with the EBL, such as upfront fees, commitment fees, or success fees payable to shareholders. These can be added by expanding the coefficient matrix to reflect their impact on each period's draw. The algebraic structure—and therefore the underlying logic of the Coefficient Matrix Method—remains the same.

5 Calculating the Equity Component – EBL & Senior Debt Funding

In this section, we consider a more generic case where Project Cost is financed through a combination of EBL and Senior Debt. The total Senior Debt Size has already been determined in Section 3. For simplicity, it is assumed that both funding sources follow a pro-rata drawdown structure, where the proportion of debt and EBL remains constant across the construction period. Upfront equity structure is discussed later int this section. The key assumptions are summarized below.

Parameter	Description
H_1, H_2, H_3	Known Hard costs in each construction period
K_1, K_2, K_3	Interest rates on Senior Debt during construction
I_1, I_2, I_3	Interest rates on the EBL during construction
L	Leverage ratio, assumed to be the same in each period under pro-rata funding
D_{total}	Total Debt known from Section 3

Again, all funding is assumed to be drawn at the end of each period, so interest applies only to previously drawn balances.

Step 1 - Formulating Periodic Project Cost, Senior Debt and EBL Drawdowns

Period 1

Cost for the Period = C_1; where

$C_1 = H_1$

$C_1 = 1 \cdot H_1 + 0 \cdot H_2 + 0 \cdot H_3$

{this is like **Equation X** in Section 4}

Funding for the Period

$D_1 = L \times C_1$

$E_1 = (1-L) \times C_1$

Period 2

Cost for the Period = C_2; where

$C_2 = D_1 \cdot K_2 + E_1 \cdot I_2 + H_2$; as interest accrues on drawn Senior Debt and EBL

substituting and rearranging this gives

$C_2 = H_1 \cdot A_2 + H_2$

where $A_2 = (L) \times K_2 + (1-L) \times I_2$

$C_2 = A_2 \cdot H_1 + 1 \cdot H_2 + 0 \cdot H_3$

{this is like **Equation Y** in section 4}

Funding for the Period

$D_2 = L \times C_2$

$E_2 = (1-L) \times C_2$

Period 3

Cost for the Period = C_3; where

$C_3 = E_1 \cdot I_3 + E_2 \cdot I_3 + D_1 \cdot K_3 + D_2 \cdot K_3 + H_3$

substituting and rearranging this gives

$C_3 = H_1 \cdot A_3 (1+A_2) + A_3 \times H_2 + H_3$

where $A_3 = (L) \times K_3 + (1-L) \times I_3$; A_2 as before

{this is like **Equation Z** in section 4}

Funding for the Period

$D_3 = L \times C_3$

$E_3 = (1-L) \times C_3$

The Periodic Project Costs are summarized in the table below

Period	Expression for Periodic Cost
1	$C_1 = 1 \cdot H_1 \quad + 0 \cdot H_2 \quad + 0 \cdot H_3$
2	$C_2 = A_2 \cdot H_1 \quad + 1 \cdot H_2 \quad + 0 \cdot H_3$
3	$C_3 = (A_3 + A_2 \cdot A_3) \cdot H_1 \quad + A_3 \cdot H_2 \quad + 1 \cdot H_3$

Further, the Periodic Costs can be written in Matrix Form as follows:

[C] = [I] x [H]; where

$$\begin{array}{ccccc} \text{Periodic Costs [C]} & & \text{Coefficient Matrix [CM]} & & \text{Hard Costs matrix [H]} \\ \begin{vmatrix} C_1 \\ C_2 \\ C_3 \end{vmatrix} & \begin{matrix} = \\ = \\ = \end{matrix} & \begin{vmatrix} 1 & 0 & 0 \\ A_2 & 1 & 0 \\ A_3(1+A_2) & A_3 & 1 \end{vmatrix} & \begin{matrix} \times \\ \times \\ \times \end{matrix} & \begin{vmatrix} H_1 \\ H_2 \\ H_3 \end{vmatrix} \end{array}$$

Again, this can be generalized for ‘**n**’ periods, where each diagonal element of the Coefficient Matrix (**CM**) equals 1, representing the direct funding of that period’s own hard cost. Every cell below the diagonal corresponds to the interest (EBL & Senior Debt on proportionate basis) for that period, multiplied by the cumulative sum of all coefficients above it. This can be summarized as below –

$$CM_{n,j} = \begin{cases} 1, & \text{if } n = j, \\ A_n \sum_{x=1}^{n-1} CM_{x,j}, & \text{if } n > j, \\ 0, & \text{if } n < j\ . \end{cases}$$

Where,
$CM_{n,j}$ = Any element of the Coefficient Matrix
n = row index
j = column index
x = summation index
A_n = (L) x K_n + (1-L) x I_n

The periodic project costs can be expressed directly in terms of the known hard costs and their associated known coefficients. Hence, periodic project costs are known at the outset. Funding of these periodic project cost can be expressed as

[D] = (L) x [C]

Periodic Debt Drawdown [D]		Periodic Costs [C]		Portion funded by Debt
[D_1]	=	[C_1]	X	L
[D_2]		[C_2]		
[D_3]		[C_3]		

[E] = (1-L) x [C]; *where*

Periodic EBL Drawdown [E]		Periodic Costs [C]		Portion funded by EBL
[E_1]	=	[C_1]	X	1-L
[E_2]		[C_2]		
[E_3]		[C_3]		

Step 2 – Calculating Leverage

As mentioned in Step 1 above

[D] = (L) x [C]

Further, [D] is known as calculated in Section 3

Expression for [C] is already known as per the matrix set up in Step 1 above.

Accordingly, L can be calculated as [D]/[C].

Step 3 – Calculating Periodic Project Cost, Senior Debt and EBL Drawdowns

Once L is calculated it can be plugged in the Periodic Debt Drawdown [D] and Periodic EBL Drawdown [E] to calculate Senior Debt and EBL drawdown in each period.

Therefore, in effect, the Senior Debt and EBL drawdown for each construction period is predetermined, which considers both i) funding of Hard costs and ii) interest during construction on the drawn Senior

Debt and EBL. As a result, when the schedule of sources and uses of funds is prepared, it naturally aligns period by period, ensuring consistency between the funding and the periodic construction cost.

Extension to Other Funding Structures

The same logic also works in more common case where equity is drawn upfront, and debt is drawn later. In such cases, each period has its own leverage L_1, L_2, L_3 ... L_n. Typically, L = 0 in early periods (as equity goes in first) & L = 1 in later periods (once equity has been fully utilized). Only in one intervening period—where both equity and debt contribute—will L take a value between 0 and 1. This is indeed a tricky problem to solve, as it requires identifying the single intermediate leverage where both debt & equity contribute during construction. However, since the total debt size is already known, this value can be determined — & this is one instance in the framework where a limited scalar Goal Seek is still required (owing to the equation's high-order nonlinearity).

Incorporating Fees and Other Financing Costs

Similar to Section 34, this method extends to include:

- Upfront fees of EBL and Senior Debt,
- Commitment fees and IDC (Interest During Construction) on both,
- Success fees etc.

All such costs can be represented by expanding the relevant coefficient matrices to reflect their proportional impact.

6 Conclusion

This article presented the Coefficient Matrix Method, a structured mathematical framework to resolve circularities in project finance models. The article demonstrated how circular dependencies that traditionally require iterative macros can instead be handled analytically (which perhaps also is inherently compatible with the future world of AI-driven financial modeling). By removing the computational delays and instability of iterative methods, this approach allows deal teams to evaluate scenarios faster and focus on strategic decision-making rather than model troubleshooting.

The proposed method was tested within a fully developed project finance model, and the methodology works effectively. That said, details matter and as every project has its own nuances — accurate application requires careful attention to assumptions, financing structure and matrix creation. We invite practitioners to test this approach, challenge it, and build on it. For, questions or feedback, please contact:

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